Week 9
MATH 34B
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21.2 Use two steps of Euler's method (ie. $\Delta t = 0.2$) for the equation $y' = y - t$ with initial condition $y(0) = 1$ to find $y(0.4)$.

$t_0 = 0$
$y_0 = 1$

$t_1 = t_0 + \Delta t = 0.2$

$y_1 = y_0 + \Delta t\, F(t_0, y_0) = 1 + 0.2 \cdot (1 - 0) = 1.2$

$t_2 = t_1 + \Delta t = 0.4$

$y_2 = y_1 + \Delta t\, F(t_1, y_1) = 1.2 + 0.2(1.2 - 0.2)$
$= 1.4$

21.3 Use Euler's method to find $y(0.4)$ if $y' = 1 - y^2$ and $y(0) = 0.5$, with a time step of 0.1.
$\Delta t = 0.1$

$t_0 = 0$
$y_0 = 0.5$

$t_1 = 0.1$

$y_1 = 0.5 + 0.1(1 - 0.5^2) = 0.575$

$t_2 = 0.2$

$y_2 = 0.575 + 0.1(1 - 0.575^2) = 0.6419375$

$t_3 = 0.3$

$y_3 = 0.6419375 + 0.1(1 - 0.6419375^2) = 0.700729125$

$t_4 = 0.4$

$y_4 = 0.700729125 + 0.1(1 - 0.700729125^2)$

21.7 A full tank initially (at t=0) contains 19 gallons. Then water is removed at a rate of 1+t gallons per minute where t is the time in minutes.

(a) How much water remains in t minutes?

(b) When (in minutes) is the tank half empty?

$\frac{dw}{dt} = 1+t$

$w = t + \frac{t^2}{2} + C$

$w(0) = 0$ ~~[illegible]~~

$\Rightarrow w = t + \frac{t^2}{2}$

w = amt. of water removed

initial, amt removed

a) $19 - \left(t + \frac{t^2}{2}\right)$

b) set $19 - \left(t + \frac{t^2}{2}\right) = \frac{19}{2}$

solve for t...

23.2 The number of items sold at a price of x dollars per item is 2000-300x. It costs 9 dollars to make the item. What price should be charged to make the most profit?

revenue = price · quantity = $x(2000-300x)$

expenses; cost = quantity · (price to make) = $9(2000-300x)$

p = profit = rev − expenses = $x(2000-300x) - 9(2000-300x)$

$p'(x) = (2000-300x)\cdot 1 + x(-300) - [9(-300)]$

set = 0, solve...

23.? In 1990 a fatal disease evolves to which 40 percent of a population of 5 million trees is susceptible. The proportion of susceptible trees which survive for a period of t years beyond 1990 is e^{-t}. How quickly is the disease killing off trees at the start of 1992? When will the population be reduced to 80 percent of the level in 1990?

60% not susceptible ⇒ 3 million

40% susceptible ⇒ 2 million.

surviving # = $\underbrace{\text{3 million}}_{\text{non-susceptible}} + \underbrace{(\text{2 million})e^{-t}}_{\text{susceptible}}$

$\Rightarrow f(t) = 3 + 2e^{-t}$

a) $f'(t) = -2e^{-t}$

plug in 2...

b). 80% ⇒ 4 million, so, set $3 + 2e^{-t} = 4$, solve for t...